# Mathematical Evaluation of the Applicability of the EEDI- Concept for RoRo- vessels

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## 1 Introduction to the Problem

Trial applications of the EEDI have shown that for some ship types there are some difficulties which make the application very challenging. Therefore, the Institute of Ship Design and Ship Safety of TU- Hamburg- Harburg was requested to analyze for which ship types the EEDI- concept can eventually not be applied and what might be the reasons. Our analysis, which is based on the mathematical evaluation of the EEDI has clearly shown that the EEDI can not be applied to more complex ship types such as RoRo- and RoPax vessels due to severe mathematical errors in the general concept. Further, our analysis can formulate a straightforward criterion which allows to decide for which ship type the EEDI can definitively not be applied.

## 2 Analysis of the EEDI- formula and baseline

In the following, we concentrate our investigation on the main contribution to EEDI, the main propulsion. This is for reasons to clearly identify the main problems, but all our findings are also valid if the remaining EEDI factors are included. Simplified, the EEDI can be expressed as follows:

$$EEDI = \frac{P \cdot SFOC \cdot CF}{DW \cdot v} \tag{1}$$

Where P denotes the MCR of the main engines, SFOC the specific fuel oil consumption (recommended as $190 g/kWh$) and CF a carbon emission factor, to be taken as $3.1144 g/CO_2/gfuel$. DW [1] is the deadweight at maximum draft, and v the speed which can be achieved at 75%MCR output of the main engine(s). The EEDI shall be below or on a prescribed baseline value, where the baseline is defined as $a \cdot DW^{-c}$, where DW is again the deadweight. a and c are coefficients that have been gained from regression analyses over existing ships.

Now the EEDI should meet the prescribed baseline value, which results in:

$$\frac{P \cdot SFOC \cdot CF}{DW \cdot v} = a \cdot DW^{-c} \tag{2}$$

[1]For some ships it was suggested to use other variables than DW to express the capacity of the ship, such as gross tonnes or lane meters, but this has no principal effect on the problems identified below.

The equation shows that the variable DW appears on both sides of the equation which makes the regression for a and c mathematically challenging, a fact we have already remarked in earlier publications.

Further, in this equation, the ship speed v is not indepent from the power P. In fact, both variables are connected by a power law which might be expressed by the well known Admirality formula in modified form:

$$P = c_P \cdot \Delta^{2/3} \cdot v^{3+k} \tag{3}$$

Here, $\Delta$ is the displacement of the ship at the draft connected to DW, $c_P$ is a coefficient and $3 + k$ is the exponent of the power law. By proper selection of $c_P$ and k, each individual speed- power curve of each individual ship can be well expressed.

It is now possible in the formula to replace the Displacement $\Delta$ by the deadweight DW and the well known ratio $DW/\Delta$. Then, the formula reads as follows:

$$P = c_P \cdot \left(\frac{DW}{\Delta}\right)^{-2/3} \cdot DW^{2/3} \cdot v^{3+k} \tag{4}$$

The reference speed for the EEDI shall be derived from 75% engine output, which leads us to:

$$0.75 \cdot P = 0.75 \cdot c_P \cdot \left(\frac{DW}{\Delta}\right)^{-2/3} \cdot DW^{2/3} \cdot v^{3+k} \tag{5}$$

Now we can replace the power $P$ in the EEDI equation 2 and solve the equation for the ship speed v. The result reads after some transformations:

$$v^{2+k} = \frac{a \cdot DW^{1-c-\frac{2}{3}} \cdot \left(\frac{DW}{\Delta}\right)^{\frac{2}{3}}}{SFOC \cdot C_F \cdot 0.75 \cdot c_P} \tag{6}$$

This equation gives the maximum ship speed which can be achieved when fulfilling the EEDI- requirement. The corresponding MCR of the main engine that can be installed into the ship can then be determined from the power law equation in equation 3. All variables in the equation are constants if we assume the the ratio $DW/\Delta$ is roughly constant for a given ship (type). In fact, this values varies slightly over the size of the ship, but this effect can at the moment be neglected to show the principal problem.

Most interesting in the formula 6 is the fact that the results do strongly depend on the sign of the DW- exponent

$$1 - c - \frac{2}{3} \tag{7}$$

where 1 comes from the EEDI- definition (1), -c comes from the baseline definition and 2/3 comes from the physically correct power- size dependeny of our power law formula.

In the exponent, the only free variable is c which comes from the baseline regression. It is now obvious that the exponent remains positive when c is smaller than 1/3. If the exponent is positive, this results in the fact that larger ships are allowed to utilize more power than smaller ships. If on the other hand c is larger than 1/3, this means that the ship is allowed to utilize more power when it becomes smaller. This is exactly the opposite.

So the value of the coefficient c triggers the most important characteristic of a speed power problem, namely the dependency of the power on the ship size, which can according to the EEDI- concept either increase or decrease with the size of the ship.

Without making any preferences to one of the two possible characteristics, it should at least turn out that this dependency is of the same type for all ship types, as they have to fulfill all the same basic physical laws. If we assume this, the regression analysis of all ship types should either show values which are clearly below 0.333, or values which are clearly above 0.333. But the latest baseline results show that this is clearly not the case. [2]

However, if we assume now that it is more reasonable when larger ships require more power compared to smaller ships, then we can clearly conclude that the EEDI- concept does definitively not work for all ship types where the regression value for c takes values which are significantly above 0.333. And if we now analyze all regression results obtained for different ship types, we find that the largest value of c is actually obtained for RoRo-vessels. For these vessels, c was computed to be about 0.7. In the past, it turned out that extreme difficulties occured for the RoRo- vessels when applying the EEDI- concept, and this is the reason why these ships have been postponed until a better solution is found.

However, our analysis has clearly shown that due to the mathematical errors we have found in the concept, it will never be possible to find a better solution fo the RoRos if the EEDI framework remains as it is. Our analysis has further developed a sound mathematical criterion for the applicability of the EEDI for certain ship types: Whenever c is above 0.333 this means that larger ships can utilize less power compared to smaller ships.

Although we have some ship types with small c values (e. g. Container Vessels where c equals 0.2), we foresee the future problem that the planned reduction of the prescribed baseline values will have an influence on the value of c, which implies the situation that the same condition as now for the RoRos will in the future appear for other ship types which have today been found uncritical with respect to the EEDI application.

[2] We refer to the latest IMO document MEPC 62/6/4 where all baseline values except for RoRos are published. For the RoRos, we refer to earlier publications, as there exists no agreed RoRo- Baseline, but all studies - e.g. DNV or EMSA/Deltamarin come to c values for RoRos about 0.7.

# 3 Application example

The above made findings are illustrated for an existing RoRo ship. Comsumptionwise, the ship is very efficient with a power law coefficient $3 + k = 3$ and $c_P = 1.872E - 3$. The deadweight of the ship is 8700 tons, the ship speed is 23 knots at a delivered power of 15700 kW. If we compute the installable power according to equations 6 and 3, we have to use the base line coefficients for RoRos, where we have taken the values of earlier IMO publications:

- a=19788, c=0.714

It could also be an option that the same ship may transport containers, and would therefore be a CONRO or container ship. In this case, also the Container Vessel baseline has to be fulfilled. For the Container Vessels, the baseline values read:

- a=156.52, c=0.200

We have computed the maximum installeble power for both vessel types as function of the deadweight, and the results are shown in the figure below.

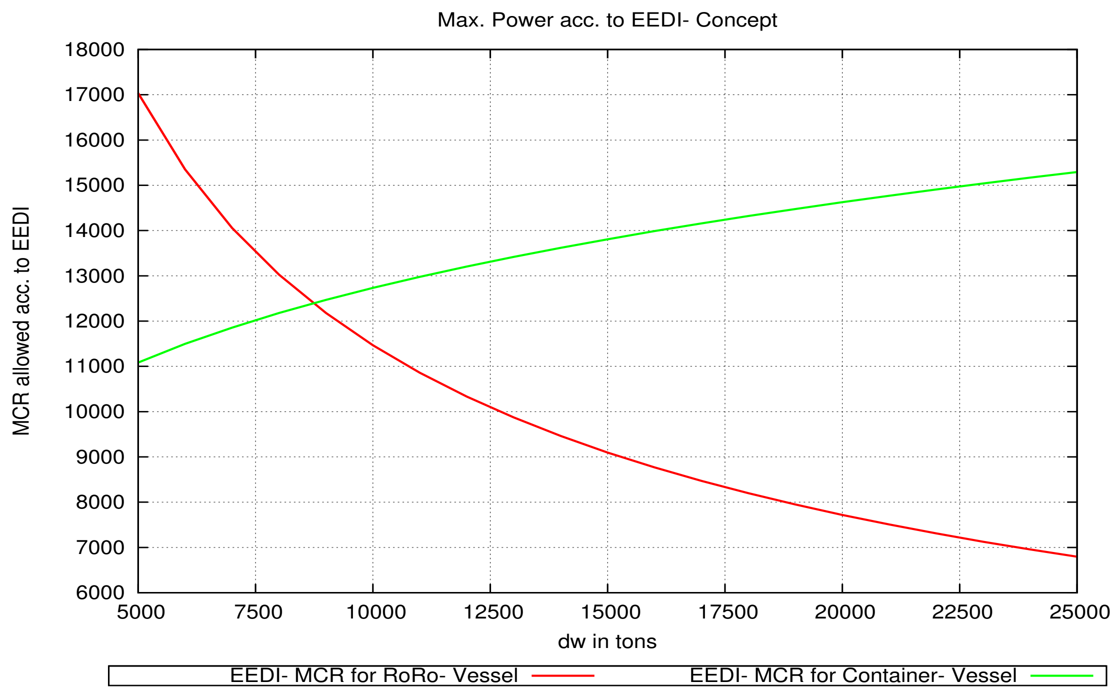


Figure 1: Maximum installable power according to the EEDI- Concept for RoRo and Container- Vessels

As expected, the two graphs show the opposite behaviour: For the reference RoRo-ship the maximum possible power that can be installed into the ship derceases with increasing ship size. For the ship designer, the fulfilment of such type of characteristic is, to put it softly, a challenging requirement. On the other hand if our reference ship would serve as container vessel - which is technically a reasonable option - the slope of the maximum installable power is exactly the opposite, which means that the bigger ship may now utilize more power. It should become clear now that these requirements are in conflict, and the related design problem can never be solved.

This leads to the conclusion that if the EEDI- concept as such in assumned to be reasonably applicable to certain ship types - which we generally doubt due to the severe mathematical errors in the concept - it is absolutely clear that the concept then can never be applied to RoRo- vessels, simply because the dependency on the ship size turns out to be exactly the opposite, although we can not find any reason why the principle physics should be different for RoRo- vessels compared to other ship types.

# 4 Conclusions

A sound mathematical analysis of the EEDI formula has shown that depending on the values of the gegression coefficient c, opposite dependecies of allowable power versus ship size can be obtained. This coincedes with the remaining value of the DW- exponent in the formula. This inconsistent behaviour of the formula is the result of two major mathematical errors which were implemented in the EEDI- concept:

- The DW appears on both sides of the EEDI equation, but is treated as independent variable in the regression for a and c
- The power P and the speed v in the left side of the EEDI equation are treated as independent varaiables, but in fact they are depending on each other.

This leads at the end to a single formula from which either the speed v or the Power P can be obtained soleley as a function of the DW of the ship, where the exponent of the DW- dependency depends on the regression results. Depending on this exponent, it is at the same time possible that a larger ship may require mor power or less power compared to a smaller ship, although the governing physical laws are the same for all ships.

The above stated errors are most relevant for RoRo- ships, where the regression value for c was found to be the highest.

It can further be concluded that a future decrease of the baseline values will modify the exponent c is such a way that it will become larger, which means that the problems that have now been agreed upon for RoRo Ships will extend to other ship types in the future.

In case the EEDI shall be applied to RoRo- vessles, we strongly recommend to work out a new base line which must then consist of really independent variables, and these are all types of variables that do not appear in the left side of the EEDI- equation.